Insight, A Study of Human Understanding by Bernard Lonergan

Notes for Session Four: 03/03/2007 by Dr. David P. Fleischacker
The Viewpoint of Arithmetic

I. The Meaning of "1"
   a. A sheep
      i. Size
      ii. Weight
      iii. height
      iv. Color
   b. A single Sheep
      i. A unified subject of characteristics
      ii. Same even in change
         1. location
         2. color
         3. size
      iii. The Residue of individuality
         1. uniqueness of data that are unified
         2. empirical residue
         3. basis of numerical 1
   c. Numerically one sheep
      i. Unity does not equal 1
         1. saying something is the subject of is not the same as saying it is 1.
      ii. Individuality does not equal unity
         1. saying something is unique is not the same as saying it is the unity of characteristics.
            a. Unity of data vs uniqueness of data
            b. **THIS** unity or **THAT** unity
         2. Math starts with distinctness, uniqueness, individualness
      iii. Individuality does not equal 1
         1. an individual thing does not equal one individual thing
            a. this sheep, this car, this house
            b. that sheep, that car, that house
      iv. From individual to 1
         1. need to "put together this and that
         2. Needs addition to mean one (needs that relation of being "put together" with another)
            a. Fido, King, Joker, and Jezebeel
            b. How many? Which one is first?

II. Rules in Arithmetic
   a. Deductive expansion: expanding one's "view" of numbers via the same operation
      i. Positive integers
      ii. Addition tables

b. Homogeneous Expansion: expanding the types of operations

Operations	**Inverse Operations**
Addition	Subtraction
→	←
Multiplication	Division
→	←
Powers	Roots
→	←

All these operations are defined ultimately in terms of addition

Subtraction: inverse of addition
Multiplication: adding a number so many times to itself.
Division: inverse of multiplication (finding the number which when added to itself so many times gets the being "divided").
Powers: multiplying a number by itself so many times
Roots: inverse of powers (finding the number which when multiplied by itself so many times gets the number being "rooted").

III. The Arithmetic Viewpoint

a. The Deductively Expanded Arithmetic Viewpoint
   i. Notice how the initial operation of addition which then gives a definition of number allows one to create a viewpoint identified as the positive integers. Another way of saying this is that it allows for the creation of the "world" of the positive integers.

b. The Homogenously Expanded Arithmetic viewpoint
   i. With the addition of subtraction, one is then able to move backward and forward in the world of the positive integers, and then add to that world the world of the negative numbers.
   ii. With the addition of the operations of multiplication, division, powers, and roots, one's viewpoint and world of positive and negative numbers not only expands in how one gets to these numbers (through these other operations), but adds to these integers the worlds of fractions (by division) and of surds (in roots).

c. Some of these activities bring questions such as what happens when one adds two negative numbers, subtracts two negative numbers, or divides a negative into a positive number. This is just the beginning of questions that will lead into the higher viewpoint of algebra.